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A NEW APPROACH FOR ESTIMATING ENTRAINMENT RATE IN CUMULUS CLOUDS

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Abstract: A new approach is presented to estimate entrainment rate in cumulus clouds with aircraft measurements. The new approach is directly derived from the definition of fractional entrainment rate and relates it to mixing fraction and the height above cloud base. The entrainment rates estimated with the new approach compare favorably with those obtained with a commonly used approach in terms of the vertical profile of entrainment rate, and are well within the current level of uncertainty. This new approach has several advantages compared to most existing methods, including elimination of the need for in-cloud measurements of temperature and water vapor content, and the potentials for straightforwardly connecting the studies of entrainment rate and microphysical effect of entrainment-mixing process, for developing a remote sensing technique to infer entrainment rates, and for investigating inhomogeneity of entrainment events.

1. Introduction

Entrainment of dry air into clouds is essential to many cloud-related processes and long-standing issues, for example, warm-rain initiation [e.g., *Beard and Ochs*, 1993; *Su et al.*, 1998; *Liu et al.*, 2002; *Lasher-Trapp et al.*, 2005; *Yum and Hudson*, 2005] and cloud feedbacks in climate models [e.g., *von Salzen and McFarlane*, 2002; *Grabowski*, 2006]. Understanding the entrainment-mixing process and improving its parameterization in climate models have attracted growing attention since the 1940s [e.g., *Arakawa and Schubert*, 1974].

A fundamental property in the study and parameterization of cumulus clouds is fractional entrainment rate (λ) [*Arakawa and Schubert*, 1974]. Several approaches for estimating λ in shallow cumulus clouds have been used in the past several decades. For example, *Stommel* [1947] estimated λ from soundings of temperature and specific humidity inside and outside of the cloud. *Betts* [1975] derived an expression that relates λ to the difference of a conserved variable between inside the cloud and the environment. Since then, similar expressions have been widely used to estimate λ from aircraft observations [e.g., *Raga et al.*, 1990; *Neggers et al.*, 2003; *Gerber et al.*, 2008] or numerical simulations [e.g., *de Rooy and Siebesma*, 2008; *Del Genio and Wu*, 2010]. *Wagner* [2011] recently developed an algorithm for retrieving λ from ground-based remote sensing observations coupled with the explicit mixing parcel model [EMPM, *Krueger et al.*, 1997; *Su et al.*, 1998]. Despite the effort and progress, the topic is still poorly understood and λ reported in literatures suffers from a wide range of uncertainties [e.g., *McCarthy*, 1974; *Neggers et al.*, 2003], hindering adequate representation of convection and clouds in atmospheric models. More efforts are needed to develop new or improve existing approaches.

Furthermore, entrainment rate and the effect of subsequent mixing processes on cloud microphysics [e.g., *Burnet and Brenguier*, 2007; *Lu et al.*, 2011]) have been largely investigated in separation [e.g., *Liu et al.*, 2002]. An approach that links the two is desirable because the two topics should be closely connected with each other.

Here a new approach is presented for estimating λ from aircraft measurements. The new approach, among other advantages, directly link the definition of λ to microphysical and thermodynamic analyses, and has the potential to directly connect the study of entrainment rate and the effects of entrainment-mixing process on cloud microphysics.

2. Formulation of the new approach

2.1 Relationship of entrainment rate to mixing fraction

The fractional entrainment rate (λ) is defined as [e.g., *Houze*, 1993]:

$$\lambda \equiv \frac{1}{m}\frac{dm}{dz}, \tag{1}$$

where m is the mass of cloud parcel and z is the vertical height above the ground. Rearrangement of Eq. (1) and integration from cloud base (z_0) to any level (z) yield

$$\int_{z_0}^{z} \lambda dz = \int_{m(z_0)}^{m(z)} \frac{dm}{m} = \ln\frac{m(z)}{m(z_0)}, \tag{2}$$

where m (z_0) and m (z) represent the mass of cloud parcel at z_0 and z, respectively. Assuming that λ is constant for the depth from z_0 to z (see Section 2.3 about the relaxation of this assumption), we obtain the expression for λ:

$$\lambda = \frac{\ln\dfrac{m(z)}{m(z_0)}}{z - z_0}. \tag{3}$$

Equation (3) can be simplified as

$$\lambda = \frac{-\ln \chi}{h}, \tag{4}$$

where $h = z\text{-}z_0$ is the height above cloud base; χ is the mixing fraction of adiabatic cloud, i.e., the mass ratio of adiabatic parcel at cloud base to the sum of adiabatic cloud parcel and entrained dry air from cloud base to the level z. Equation (4) reveals that the key to obtaining λ lies in estimating χ.

2.2 Estimation of mixing fraction χ

The mixing fraction χ can be estimated based on the conservations of total water and energy during the isobaric mixing process when the environmental air at the observation level is assumed to be entrained into adiabatic cloud [*Burnet and Brenguier*, 2007; *Gerber et al.*, 2008; *Krueger*, 2008; *Lehmann et al.*, 2009]. The equations are:

$$q_{\mathrm{L}} + q_{\mathrm{vs}}(T) = \chi[q_{\mathrm{vs}}(T_{\mathrm{a}}) + q_{\mathrm{La}}] + (1-\chi)q_{\mathrm{ve}} \tag{5a}$$

$$c_{\mathrm{p}}T = c_{\mathrm{p}}T_{\mathrm{a}}\chi + c_{\mathrm{p}}T_{\mathrm{e}}(1-\chi) - L_{\mathrm{v}}(q_{\mathrm{La}}\chi - q_{\mathrm{L}}) \tag{5b}$$

$$q_{\mathrm{vs}}(T) = 0.622\frac{e_{\mathrm{s}}(T)}{p - e_s(T)} \tag{5c}$$

$$q_{\mathrm{vs}}(T_{\mathrm{a}}) = 0.622\frac{e_{\mathrm{s}}(T_{\mathrm{a}})}{p - e_s(T_{\mathrm{a}})} \tag{5d}$$

where T, $q_{\mathrm{vs}}(T)$ and q_{L} are the temperature, saturated water vapor mixing ratio, liquid water mixing ratio, respectively, when the mixing process is finished and a new saturation is achieved; T_{a}, $q_{\mathrm{vs}}(T_{\mathrm{a}})$ and q_{La} are the temperature, saturated water vapor mixing ratio, adiabatic liquid water mixing ratio in adiabatic cloud, respectively; T_{e}, q_{ve} are temperature and water vapor mixing ratio of the entrained dry air, respectively; e_{s} is saturated water vapor pressure; c_{p} is specific heat capacity at constant pressure; p is air pressure; L_{v} is latent heat. These or similar equations are the basis for generating a mixing diagram widely used in the study of entrainment-mixing mechanisms [e.g., *Burnet and Brenguier*, 2007; *Gerber et al.*, 2008].

Assuming the cloud observed by aircraft has achieved saturation, the liquid water mixing ratio in cloud is q_L. Based on q_L and along with T_a, $q_{vs}(T_a)$ and q_{La} in adiabatic cloud as well as T_e, q_{ve} in the environment, χ can be obtained by solving this set of equations. It is noteworthy that the observed values of T and $q_{vs}(T)$ in cloud are not used during the calculation of χ.

2.3. Vertical profile of average entrainment rate

In real measurements, a number of data points are collected at a flight level, with the exact number depending on the flight length and data sampling frequency. Application of the new approach allows us to obtain λ for all the data points in question. The average λ for each level can be obtained by averaging all the values of λ at the same sampling level.

Note that λ from Eq. (4) actually represents an effective entrainment rate between z_0 and z. An adjustment is needed to obtain the profile of entrainment rate. Briefly, if aircraft penetrates n horizontal levels to collect the data and the average λ for the j-th level is labeled as λ_{aj} (j =1, 2,⋯, n), the adjusted values (λ_{pj}, j=1, 2,⋯, n) are given by

$$\lambda_{pj} = \frac{\lambda_{aj} h_j - \lambda_{a(j-1)} h_{j-1}}{h_j - h_{j-1}} (\text{j} = 2, 3, \cdots, \text{n}), \tag{6a}$$

$$\lambda_{pj} = \lambda_{aj} (\text{j} = 1). \tag{6b}$$

The adjusted j-th height is given by $(h_j + h_{j-1})/2$ for j=2, 3,⋯, n and $h_j/2$ for j=1. The adjustment follows from the assumption that the total entrained mass between any two levels are evenly distributed at the mid-level between the two heights. It is obvious that as most existing approaches, the profile accuracy is expected to increase with increasing vertical sampling resolution.

3. Validation

Gerber et al. [2008] estimated λ in a trade-wind cumulus case (RF 12) observed with the NCAR C-130 research aircraft during the RICO (Rain in Cumulus over the Ocean) project [*Rauber et al.*, 2007] using an approach similar to *Betts* [1975] with total water mixing ratio as the conserved property. To validate our approach, we apply it to the same cloud, and compare the results with λ obtained by *Gerber et al.* [2008]. In this study, we use the 25 Hz data of liquid water content, temperature and water vapor mixing ratio, measured by Particle Volume Monitor (PVM) probe, Rosemount sensor and Lyman-Alpha hygrometer, respectively. Cloud base is determined by fitting the peak liquid water content values with a linear profile [*Lehmann et al.*, 2009]. Based on the height and temperature at the cloud base, which is taken from the aircraft sounding value at the height of cloud base [*McCarthy*, 1974], we calculated the T_a, $q_{vs}(T_a)$ and q_{La} in the adiabatic cloud parcel at the five observation levels, respectively (Table 1). The T_e and relative humidity in the environmental air that entrained into the adiabatic cloud at the five observation levels are taken from the aircraft sounding values at these levels [*McCarthy*, 1974; *Lehmann et al.*, 2009]. Based on the properties in Table 1 and q_L observed in cloud, entrainment rates are calculated.

As shown in Fig. 1a, the average entrainment rates from the cloud base to the five sampling levels are in the range of 0.75-1.60 km^{-1}. Figure 1b further compares the vertical profile of entrainment rate obtained from the new approach with that reported by *Gerber et al.* [2008]. The vertical variation trends are very similar, although the values of our result are smaller. However, the average entrainment rate from this approach (1.2 km^{-1}) is close to the results reported by

Raga et al. [1990] (1.30 km^{-1}) and by *Neggers et al.* [2003] (1.37 km^{-1}), both using an approach similar to *Gerber et al.* [2008] with total water mixing ratio as the conserved property. *Wagner* [2011] also reported an average of 1.02 km^{-1} using a ground-based remote sensing technique. *McCarthy et al.* [1974] estimated fractional entrainment rate by finding the best match of temperature or liquid water content from mixing models and aircraft measurements. The results were different between using temperature and liquid water content: the former was in the range of 0.17-1.20 km^{-1} with the mean of 0.63 km^{-1} and the latter was in the range of 0.26-7.39 km^{-1} with the mean of 1.53 km^{-1}. So both the individual entrainment rates at different levels and the average value of all levels from the new approach are well within the range of existing estimates. Such a favorable agreement lends credence to the new approach.

Furthermore, a combination of the results shown in Fig. 1 and Eq. (4) suggests that the minimum of λ in the middle of the cloud results from a competition between the increasing height above the cloud base (h) and the increase in mixing fraction of dry air ($1-\chi$) with the increasing height.

4. Summary

A new approach is presented for estimating fractional entrainment rate in cumulus clouds. This new approach is derived from the definition of fractional entrainment rate and relates the entrainment rate to the mass ratio of the adiabatic cloud parcel to the mixed cloud parcel affected by entrainment process. It is further shown that the mass ratio can be determined by considering the conservations of total water and energy during the entrainment-mixing process. The new approach is validated by comparing the inferred entrainment rates in a cumulus cloud against those estimated using a traditional approach. The comparison shows encouraging agreements. Furthermore, the estimated entrainment rates at different levels and their average all fall within the range of existing estimates.

Compared to most existing approaches, the new approach at least has four advantages. First, the approach reveals a potential connection via mixing fraction between the two aspects of entrainment-mixing process: fractional entrainment dynamics and microphysical analysis, which have been studied largely in separation. For example, the reaction time after dry air is entrained into cloud is critical for determining mixing mechanisms [*Lehmann et al.*, 2009; *Lu et al.*, 2011]. However, the effect of mixing fraction on reaction time has not been considered. Therefore, introduction of mixing fraction in the calculation of reaction time can link the analysis of mixing mechanisms to entrainment rate as estimated by our approach. Second, it is known that measurements of temperature and water vapor mixing ratio in cloud are difficult and problematic [*Burnet and Brenguier*, 2007]. The new approach removes this concern because it only requires liquid water mixing ratio in cloud, temperature and water vapor mixing ratio in environment and at cloud base height. Third, the basic principles underlying the approach can be extended to developing a remote-sensing based approach, which enables long-term observations. Finally, most existing methods are for obtaining entrainment rates averaged over long distances, which prohibits investigation of horizontal inhomogeneity of entrainment events. Instead, the new approach is applicable to cloud "parcels" as small as ~ 4 meters for the cloud examined here (25 Hz sampling rate and ~ 100 ms^{-1} of aircraft speed) or even smaller depending on observation frequency. A recent modeling study suggested that inhomogeneous entrainment rate is a major factor for determining cloud structure [*Romps and Kuang*, 2010]. We plan to apply the new approach to examine the horizontal inhomogeneity of entrainment events.

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Table 1. Summary of key variables at the five sampling altitudes

Level	Height above Cloud Base (m)	Temperature in the Adiabatic Cloud (°C)	Water Vapor Mixing Ratio in the Adiabatic Cloud (g kg^{-1})	Liquid Water Mixing Ratio in the Adiabatic Cloud (g kg^{-1})	Temperature in the Environment (°C)	Relative Humidity in the Environment (%)
1	261.9	20.0	16.0	0.59	19.3	87.4
2	448.7	19.2	15.6	1.01	18.2	85.9
3	622.8	18.5	15.2	1.40	17.2	83.3
4	933.1	17.2	14.5	2.09	15.7	72.5
5	1088.1	16.5	14.1	2.43	14.9	80.6

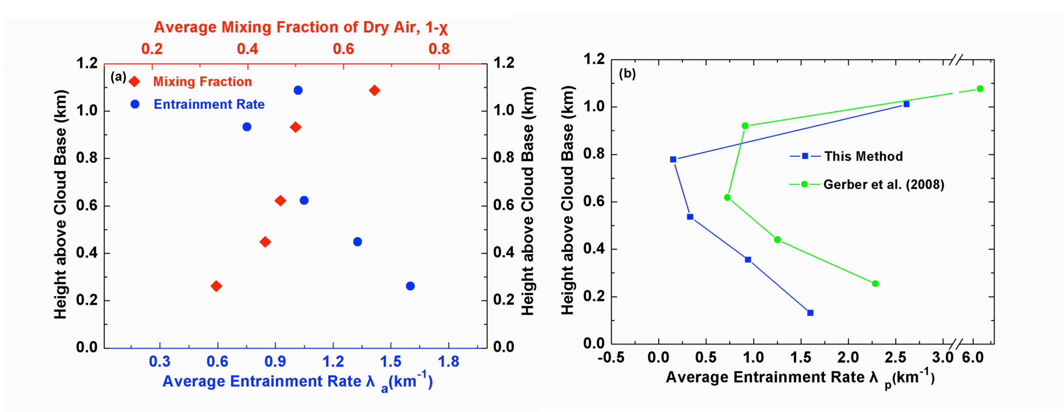

Figure 1. (a) Average entrainment rates (λ_{aj}, j =1, 2,⋯, 5) from the cloud base to the five observation levels and corresponding mixing fractions of entrained dry air at the five levels. (b) Comparison of the vertical profile of entrainment rates (λ_{pj}, j =1, 2,⋯, 5) obtained using the new approach and the result from a traditional approach reported by *Gerber et al.* [2008].